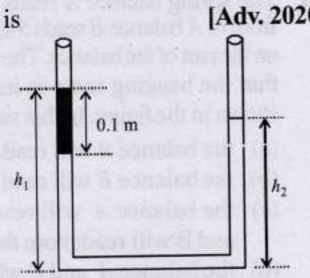
Chapter
Mechanical Properties of Fluids

Topic-1: Pressure, Density, Pascal's Law and Archimedes' Principle

1 MCQs with One Correct Answer

1. An open-ended U-tube of uniform cross-sectional area contains water (density 10^3 kg m^{-3}). Initially the water level stands at 0.29 m from the bottom in each arm. Kerosene oil (a water-immiscible liquid) of density 800 kg m^{-3} is added to the left arm until its length is 0.1 m, as shown in the schematic figure below. The ratio $\left(\frac{h_1}{h_2}\right)$ of the heights of the liquid in the two arms is **[Adv. 2020]**



(a) $\frac{15}{14}$ (b) $\frac{35}{33}$

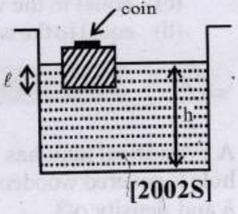
(c) $\frac{7}{6}$ (d) $\frac{5}{4}$

2. A uniform cylinder of length L and mass M having cross-sectional area A is suspended, with its length vertical, from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is: **[2012]**

(a) $\frac{Mg}{k}$ (b) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

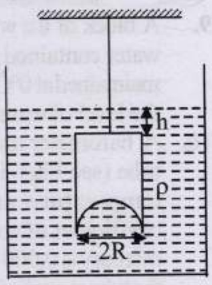
(c) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (d) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

3. A wooden block, with a coin placed on its top, floats in water as shown in figure. The distance ℓ and h are shown here. After some time the coin falls into the water. Then **[2002S]**



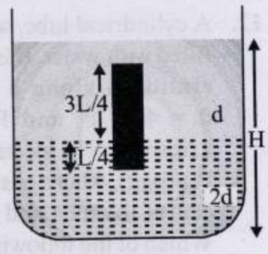
(a) ℓ decreases and h increases
(b) ℓ increases and h decreases
(c) both ℓ and h increase
(d) both ℓ and h decrease

4. A hemispherical portion of radius R is removed from the bottom of a cylinder of radius R. The volume of the remaining cylinder is V and its mass M. It is suspended by a string in a liquid of density ρ where it stays vertical. The upper surface of the cylinder is at a depth h below the liquid surface. The force on the bottom of the cylinder by the liquid is



(a) Mg (b) $Mg - V\rho g$ **[2001S]**
(c) $Mg + \pi R^2 h\rho g$ (d) $\rho g(V + \pi R^2 h)$

5. A homogeneous solid cylinder of length L $(L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the figure. The lower density liquid is open to atmosphere having pressure P_0. Then density D of solid is given by **[1995S]**



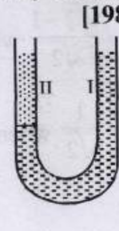
(a) $\frac{5}{4}d$

(b) $\frac{4}{5}d$

(c) $4d$

(d) $\frac{d}{5}$

6. A U-tube of uniform cross section (see Fig) is partially filled with a liquid I. Another liquid II which does not mix with liquid I is poured into one side. It is found that the liquid levels of the two sides of the tube are the same, while the level of liquid I has risen by 2 cm. If the specific gravity of liquid I is 1.1, the specific gravity of liquid II must be **[1983 - 1 Mark]**



(a) 1.12
(b) 1.1
(c) 1.05
(d) 1.0

3 Numeric Answer

7. A cubical solid aluminium (bulk modulus $= -V\frac{dP}{dV}$ = 70 GPa) block has an edge length of 1 m on the surface of the earth. It is kept on the floor of a 5 km deep ocean. Taking the average density of water and the acceleration due to gravity to be 10^3 kg m^{-3} and 10 ms^{-2}, respectively, the change in the edge length of the block in mm is _____. **[Adv. 2020]**

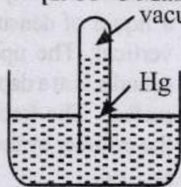
4 Fill in the Blanks

8. A solid sphere of radius R made of a material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless pistion of area A floats on the surface of the liquid. When a mass M is placed on the piston to c o m p r e s s the liquid the fractional change in the radius of the sphere, $\delta R/R$, is .... **[1988 - 2 Mark]**

5 True / False

9. A block of ice with a lead shot embedded in it is floating on water contained in a vessel. The temperature of the system is maintained at 0°C as the ice melts. When the ice melts completely the level of water in the vessel rises. **[1986 - 3 Marks]**

10. A barometer made of a very narrow tube (see Fig) is placed at normal temperature and pressure. The coefficient of volume expansion of mercury is 0.00018 per C° and that of the tube is negligible. The temperature of mercury in the barometer is now raised by 1°C, but the temperature of the atmosphere does not change. Then the mercury height in the tube remains unchanged. **[1983 - 2 Marks]**



11. A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, the level of the water in the pond decreases. **[1980]**

6 MCQs with One or More than One Correct Answer

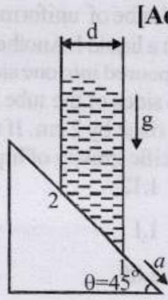
12. A cylindrical tube, with its base as shown in the figure, is filled with water. It is moving down with a constant acceleration a along a fixed inclined plane with angle $\theta = 45°$. P_1 and P_2 are pressures at points 1 and 2, respectively, located at the base of the tube. Let $\beta = (P_1 - P_2)/(\rho g d)$, where ρ is density of water, d is the inner diameter of the tube and g is the acceleration due to gravity. Which of the following statement(s) is(are) correct ? **[Adv. 2021]**

(a) $\beta = 0$ when $a = g/\sqrt{2}$

(b) $\beta > 0$ when $a = g/\sqrt{2}$

(c) $\beta = \frac{\sqrt{2}-1}{\sqrt{2}}$ when $a = g/2$

(d) $\beta = \frac{1}{\sqrt{2}}$ when $a = g/2$



13. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If $P(r)$ is the pressure at $r(r < R)$, then the correct option(s) is (are) **[Adv. 2015]**

(a) $P(r=0)=0$

(b) $\frac{P(r=3R/4)}{P(r=2R/3)} = \frac{63}{80}$

(c) $\frac{P(r=3R/5)}{P(r=2R/5)} = \frac{16}{21}$

(d) $\frac{P(r=R/2)}{P(r=R/3)} = \frac{20}{27}$

14. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are) **[Adv. 2013]**

(a) The net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

(b) The net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(c) The light sphere is partially submerged

(d) The light sphere is completely submerged

15. A vessel contains oil (density = 0.8 gm/cm^3) over mercury (density = 13.6 gm cm^3). A homogeneous sphere floats with half its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm/cm^3 is **[1988 - 2 Mark]**

(a) 3.3 (b) 6.4 (c) 7.2 (d) 12.8

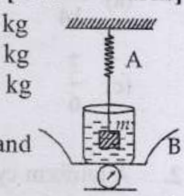
16. The spring balance A reads 2 kg with a block m suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in the figure. In this situation: **[1985 - 2 Marks]**

(a) the balance A will read more than 2 kg

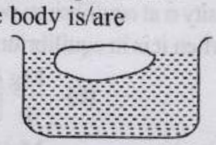
(b) the balance B will read more than 5 kg

(c) the balance A will read less than 2 kg and B will read more than 5 kg

(d) the balance A and B will read 2 kg and 5 kg respectively



17. A body floats in a liquid contained in a beaker. The whole system as shown in Figure falls freely under gravity. The upthrust on the body is/are **[1982 - 3 Marks]**



(a) zero

(b) equal to the weight of the liquid displaced

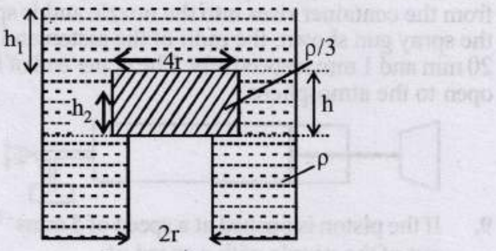
(c) equal to the weight of the body in air

(d) equal to the weight of the immersed portion of the body

8 Comprehension/Passage Based Questions

Passage

A cylindrical tank has a hole of diameter $2r$ in its bottom. The hole is covered wooden cylindrical block of diameter $4r$, height h and density $\rho/3$.



Situation I : Initially, the tank is filled with water of density ρ to a height such that the height of water above the top of the block is h_1 (measured from the top of the block).

Situation II : The water is removed from the tank to a height h_2 (measured from the bottom of the block), as shown in the figure. The height h_2 is smaller than h (height of the block) and thus the block is exposed to the atmosphere.

18. Find the minimum value of height h_1 (in situation 1), for which the block just starts to move up? **[2006 - 5M, –2]**

(a) $\frac{2h}{3}$ (b) $\frac{5h}{4}$ (c) $\frac{5h}{3}$ (d) $\frac{5h}{2}$

19. Find the height of the water level h_2 (in situation 2), for which the block remains in its original position without the application of any external force **[2006 - 5M, –2]**

(a) $\frac{h}{3}$ (b) $\frac{4h}{9}$

(c) $\frac{2h}{3}$ (d) h

20. In situation 2, if h_2 is further decreased, then **[2006 - 5M, –2]**

(a) cylinder will not move up and remains at its original position

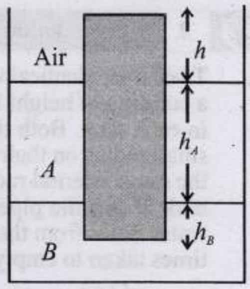
(b) for $h_2 = \frac{h}{3}$, cylinder again starts moving up

(c) for $h_2 = \frac{h}{4}$, cylinder again starts moving up

(d) for $h_2 = \frac{h}{5}$, cylinder again starts moving up

10 Subjective Problems

21. A uniform solid cylinder of density 0.8 g/cm^3 floats in equilibrium in a combination of two non-mixing liquids A and B with its axis vertical.



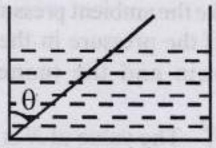
The densities of the liquids A and B are 0.7 g/cm^3 and 1.2 g/cm^3, respectively. The height of liquid A is $h_A = 1.2$ cm. The length of the part of the cylinder immersed in liquid B is $h_B = 0.8$ cm. **[2002 - 5 Marks]**

(a) Find the total force exerted by liquid A on the cylinder.

(b) Find h, the length of the part of the cylinder in air.

(c) The cylinder is depressed in such a way that its top surface is just below the upper surface of liquid A and is then released. Find the acceleration of the cylinder immediately after it is released.

22. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in fig The tank is filled with water upto a height 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0°) **[1984- 8 Marks]**



23. A boat floating in a water tank is carrying a number of large stones. If the stones are unloaded into water, what will happen to the water level? **[1979]**

24. A cube of wood supporting 200 gm mass just floats in water. When the mass is removed, the cube ruses by 2cm. What is the size of the cube? **[1978]**

25. A column of mercury of 10 cm length is contained in the middle of a narrow horizontal 1 m long tube which is closed at both the ends. Both the halves of the tube contain air at a pressure of 76 cm of mercury. By what distance will the column of mercury be displaced if the tube is held vertically? **[1978]**

Topic-2: Fluid Flow, Reynold's Number and Bernoulli's Principle

1 MCQs with One Correct Answer

1. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg) **[2012]**

(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

2. Water is filled in a container upto height 3m. A small hole of area 'a' is punched in the wall of the container at a height 52.5 cm from the bottom. The cross sectional area of the container is A. If $a/A = 0.1$ then v^2 is (where v is the velocity of water coming out of the hole) **[2005S]**

(a) 50 (b) 51 (c) 48 (d) 51.5

3. A large open tank has two holes in the wall. One is a square hole of side L at a depth y from the top and the other is a circular hole of radius R at a depth $4y$ from the top. When the tank is completely filled with water, the quantities of water flowing out per second from both holes are the same.

Then, R is equal to **[2000S]**

(a) $\frac{L}{\sqrt{2\pi}}$ (b) $2\pi L$ (c) L (d) $\frac{L}{2\pi}$

2 Integer Value Answer

4. Two large, identical water tanks, 1 and 2, kept on the top of a building of height H, are filled with water up to height h in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows from the tanks 1 and 2 through the holes, the times taken to empty the tanks are t_1 and t_2, respectively.

If $H = \left(\frac{16}{9}\right)h$, then the ratio $\frac{t_1}{t_2}$ is ______. **[Adv. 2024]**

5. Water from a tap emerges vertically downwards with an initial spped of 1.0 m s^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water, and that the flow is steady. The cross-sectional area of the stream 0.15 m below the tap is **[1998S - 2 Marks]**

(a) $5.0 \times 10^{-4} \text{m}^2$ (b) $1.0 \times 10^{-5} \text{m}^2$
(c) $5.0 \times 10^{-5} \text{m}^2$ (d) $2.0 \times 10^{-5} \text{m}^2$

6. A train with cross-sectional area S_t is moving with speed v_t inside a long tunnel of cross-sectional area S_0 ($S_0 = 4S_t$). Assume that almost all the air (density ρ) in front of the train flows back between its sides and the walls of the tunnel. Also, the air flow with respect to the train is steady and laminar. Take the ambient pressure and that inside the train to be p_0. If the pressure in the region between the sides of the train and the tunnel walls is p, then

$p_0 - p = \frac{7}{2N}\rho v_t^2$. The value of N is ______.

7. A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height (in mm) of water level due to opening of the orifice.

[Take atmospheric pressure $= 1.0 \times 10^5 \text{ N/m}^2$, density of water $= 1000 \text{ kg/m}^3$ and $g = 10 \text{ m/s}^2$. Neglect any effect of surface tension.] **[2009]**

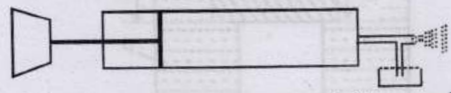
4 Fill in the Blanks

8. A horizontal pipeline carries water in a streamline flow. At a point along the pipe, where the cross- sectional area is 10 cm^2, the water velocity is 1 ms^{-1} and the pressure is 2000 Pa. The pressure of water at another point where the cross-sectional area is 5 cm^2, is... Pa. (Density of water $= 10^3 \text{ kg.m}^{-3}$) **[1994 - 2 Marks]**

8 Comprehension/Passage Based Questions

Passage

A spray gun is shown in the figure where a piston pushes air out of a nozzle. A thin tube of uniform cross section is connected to the nozzle. The other end of the tube is in a small liquid container. As the piston pushes air through the nozzle, the liquid from the container rises into the nozzle and is sprayed out. For the spray gun shown, the radii of the piston and the nozzle are 20 mm and 1 mm respectively. The upper end of the container is open to the atmosphere.



9. If the piston is pushed at a speed of 5 mms^{-1}, the air comes out of the nozzle with a speed of **[Adv. 2014]**

(a) 0.1 ms^{-1} (b) 1 ms^{-1} (c) 2 ms^{-1} (d) 8 ms^{-1}

10. If the density of air is ρ_a, and that of the liquid ρ_l, then for a given piston speed the rate (volume per unit time) at which the liquid is sprayed will be proportional to **[Adv. 2014]**

(a) $\sqrt{\frac{\rho_a}{\rho_l}}$ (b) $\sqrt{\rho_a \rho_l}$ (c) $\sqrt{\frac{\rho_l}{\rho_a}}$ (d) ρ_l

9 Assertion and Reason Type Questions

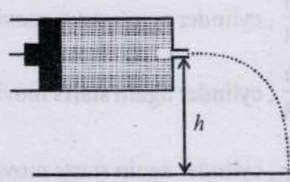
11. **STATEMENT-1 :** The stream of water flowing at high speed from a garden hose pipe tends to spread like a fountain when held vertically up, but tends to narrow down when held vertically down.

STATEMENT-2 : In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant. **[2008]**

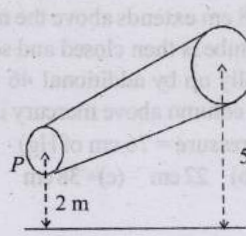
(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True

10 Subjective Problems

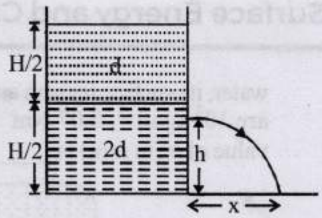
12. A tube has two area of cross-sections as shown in figure. The diameters of the tube are 8 mm and 2 mm. Find range of water falling on horizontal surface, if piston is moving with a constant velocity of 0.25 m/s, $h = 1.25$ m ($g = 10 \text{ m/s}^2$) **[2004 - 2 Marks]**



13. A non-viscous liquid of constant density 1000 kg/m^3 flows in a streamline motion along a tube of variable cross section. The tube is kept inclined in the vertical plane as shown in Figure. The area of cross section of the tube two points P and Q at heights of 2 metres and 5 metres are respectively $4 \times 10^{-3} \text{ m}^2$ and $8 \times 10^{-3} \text{ m}^2$. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. **[1997 - 5 Marks]**



14. A container of large uniform cross-sectional area A resting on a horizontal surface, holds two immiscible, non-viscous and incompressible liquids of densities d and $2d$, each of height $H/2$ as shown in the figure. The lower density liquid is open to the atmosphere having pressure P_0. **[1995 - 5 + 5 Marks]**



(a) A homogeneous solid cylinder of length $L(L < H/2)$, cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid. Determine:

(i) the density D of the solid and

(ii) the total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area $s(s << A)$ is punched on the vertical side of the container at a height $h(h < H/2)$. Determine :

(i) the initial speed of efflux of the liquid at the hole,

(ii) the horizontal distance x travelled by the liquid initially, and

(iii) the height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m.

(Neglect the air resistance in these calculations.)

Topic-3: Viscosity and Terminal Velocity

3 Numeric Answer

1. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is **[Adv. 2016]**

6 MCQs with One or More than One Correct Answer

2. A table tennis ball has radius $\left(\frac{3}{2}\right)\times10^{-2}$ m and mass $\left(\frac{22}{7}\right)\times10^{-3}$ kg. It is slowly pushed down into a swimming pool to a depth of $d = 0.7$ m below the water surface and then released from rest. It emerges from the water surface at speed v, without getting wet, and rises up to a height H. Which of the following option(s) is(are) correct?

[Given: $\pi = \frac{22}{7}$, $g = 10\ ms^{-2}$, density of water $= 1 \times 10^3$ kg m^{-3}, viscosity of water $= 1 \times 10^{-3}$ Pa-s.] **[Adv. 2024]**

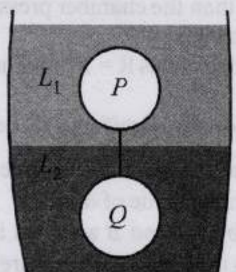
(a) The work done in pushing the ball to the depth d is 0.077 J.

(b) If we neglect the viscous force in water, then the speed $v = 7$ m/s.

(c) If we neglect the viscous force in water, then the height H = 1.4 m.

(d) The ratio of the magnitudes of the net force excluding the viscous force to the maximum viscous force in water is $\frac{500}{9}$.

3. Two spheres P and Q of equal radii have densities ρ_1 and ρ_2, respectively. The spheres are connected by a massless string and placed in liquids L_1 and L_2 of densities σ_1 and σ_2 and viscosities η_1 and η_2, respectively. They float in equilibrium with the sphere P in L_1 and sphere Q in L_2 and the string being taut (see figure). If sphere P alone in L_2 has terminal velocity $\vec{V}_P$ and Q alone in L_1 has terminal velocity $\vec{V}_Q$, then **[Adv. 2015]**



(a) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_1}{\eta_2}$

(b) $\frac{|\vec{V}_P|}{|\vec{V}_Q|} = \frac{\eta_2}{\eta_1}$

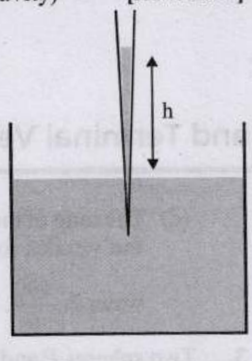
(c) $\vec{V}_P \cdot \vec{V}_Q > 0$

(d) $\vec{V}_P \cdot \vec{V}_Q < 0$

Topic-4: Surface Tension, Surface Energy and Capillarity

1 MCQs with One Correct Answer

1. A glass capillary tube is of the shape of a truncated cone with an apex angle α so that its two ends have cross sections of different radii. When dipped in water vertically, water rises in it to a height h, where the radius of its cross section is b. If the surface tension of water is S, its density is ρ, and its contact angle with glass is θ, the value of h will be (g is the acceleration due to gravity) **[Adv. 2014]**



(a) $\frac{2S}{b\rho g}\cos(\theta-\alpha)$

(b) $\frac{2S}{b\rho g}\cos(\theta+\alpha)$

(c) $\frac{2S}{b\rho g}\cos(\theta-\alpha/2)$

(d) $\frac{2S}{b\rho g}\cos(\theta+\alpha/2)$

2 Integer Value Answer

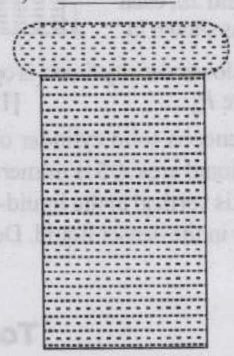
2. A spherical soap bubble inside an air chamber at pressure $P_0 = 10^5$ Pa has a certain radius so that the excess pressure inside the bubble is $\Delta P = 144$ Pa. Now, the chamber pressure is reduced to $8P_0/27$ so that the bubble radius and its excess pressure change. In this process, all the temperatures remain unchanged. Assume air to be an ideal gas and the excess pressure ΔP in both the cases to be much smaller than the chamber pressure. The new excess pressure ΔP in Pa is ______. **[Adv. 2024]**

3. A drop of liquid of radius $R = 10^{-2}$ m having surface tension $S = \frac{0.1}{4\pi}\,\text{Nm}^{-1}$ divides itself into K identical drops. In this process the total change in the surface energy $\Delta U = 10^{-3}$ J. If $K = 10^{\alpha}$ then the value of α is **[Adv. 2017]**

4. Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m^2. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio n_B/n_A, where n_A and n_B are the number of moles of air in bubbles A and B, respectively. [Neglect the effect of gravity.] **[2009]**

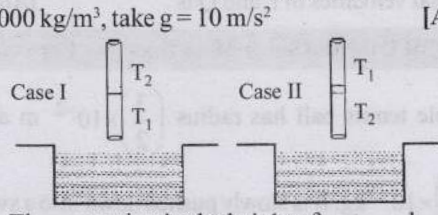
3 Numeric Answer

5. When water is filled carefully in a glass, one can fill it to a height h above the rim of the glass due to the surface tension of water. To calculate h just before water starts flowing, model the shape of the water above the rim as a disc of thickness h having semicircular edges, as shown schematically in the figure. When the pressure of water at the bottom of this disc exceeds what can be withstood due to the surface tension, the water surface breaks near the rim and water starts flowing from there. If the density of water, its surface tension and the acceleration due to gravity are 10^3 kg m^{-3}, 0.07 Nm^{-1} and 10 ms^{-2}, respectively, the value of h (in mm) is ______. **[Adv. 2020]**



6 MCQs with One or More than One Correct Answer

6. A cylindrical capillary tube of 0.2mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0° and 60°, respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct? [surface tension of water = 0.075 N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2] **[Adv. 2019]**



(a) The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus, will be different for both cases.

(b) For case II, if the capillary joint is 5cm above the water surface, the height of water column raised in the tube will be 3.75cm. (Neglect the weight of the water in the meniscus).

(c) For case I, if the joint is kept at 8cm above the water surface, the height of water column in the tubne will be 7.5cm. [Neglect the weight of the water in the meniscus]

(d) For case I, if the capillary joint is 5cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. [Neglect the weight of the water in the meniscus]

7. A uniform capillary tube of inner radius r is dipped vertically into a beaker filled with water. The water rises to a height h in the capillary tube above the water surface in the beaker. The surface tension of water is σ. The angle of contact between water and the wall of the capillary tube is θ. Ignore the mass of water in the meniscus. Which of the following statements is (are) true? **[Adv. 2018]**

(a) For a given material of the capillary tube, h decreases with increase in r

(b) For a given material of the capillary tube, h is independent of σ

(c) If this experiment is performed in a lift going up with a constant acceleration, then h decreases

(d) h is proportional to contact angle θ

8 Comprehension/Passage Based Questions

Passage

When liquid medicine of density ρ is to put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When this force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

8. If the radius of the opening of the dropper is r, the vertical force due to the surface tension on the drop of radius R (assuming $r << R$) is **[2010]**

(a) $2\pi rT$ (b) $2\pi RT$ (c) $\frac{2\pi r^2T}{R}$ (d) $\frac{2\pi R^2T}{r}$

9. If $r = 5\times10^{-4}$ m, $\rho = 10^3$ kgm^{-3}, $g = 10$ ms^{-2}, $T = 0.11$ Nm^{-1}, the radius of the drop when it detaches from the dropper is approximately **[2010]**

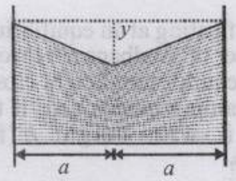
(a) 1.4×10^{-3} m (b) 3.3×10^{-3} m
(c) 2.0×10^{-3} m (d) 4.1×10^{-3} m

10. After the drop detaches, its surface energy is **[2010]**

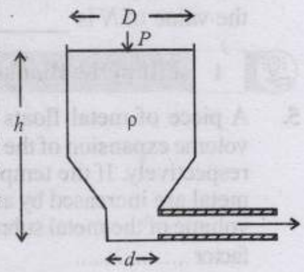
(a) 1.4×10^{-6} J (b) 2.7×10^{-6} J
(c) 5.4×10^{-6} J (d) 8.1×10^{-6} J

10 Subjective Problems

11. A uniform wire having mass per unit length λ is placed over a liquid surface. The wire causes the liquid to depress by $y (y << a)$ as shown in figure. Find surface tension of liquid. Neglect end effect. **[2004 - 2 Marks]**

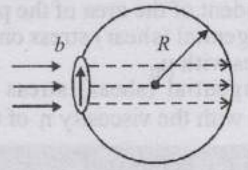


12. Shown in the figure is a container whose top and bottom diameters are D and d respectively. At the bottom of the container, there is a capillary tube of outer radius b and inner radius a. The volume flow rate in the capillary is Q.



If the capillary is removed the liquid comes out with a velocity of v_0. The density of the liquid is given as ρ. Calculate the coefficient of viscosity η. **[2003 - 4 Marks]**

13. A bubble having surface tension T and radius R is formed on a ring of radius b ($b << R$). Air is blown inside the tube with velocity v as shown. The air molecule collides perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring. **[2003 - 4 Marks]**



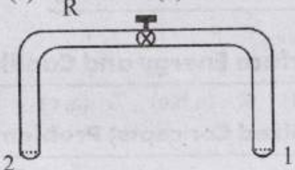
Topic-5: Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to **[Adv. 2017]**

(a) R (b) R^3 (c) $\frac{1}{R}$ (d) $R^{2/3}$

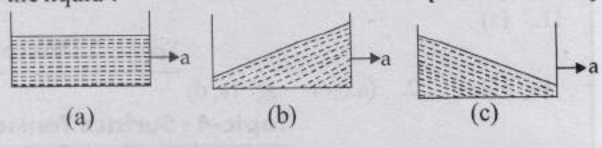
2. A glass tube of uniform internal radius (r) has a valve separating the two identical ends. Initially, the valve is in a tightly closed position.



End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve, **[2008]**

(a) air from end 1 flows towards end 2. No change in the volume of the soap bubbles
(b) air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases
(c) no changes occurs
(d) air from end 2 flows towards end 1. volume of the soap bubble at end 1 increases

3. A vessel containing water is given a constant acceleration 'a' towards the right along a straight horizontal path. Which of the following diagrams in Fig. represents the surface of the liquid ? **[1981- 2 Marks]**



2 Integer Value Answer

4. A hot air balloon is carrying some passengers, and a few sandbags of mass 1 kg each so that its total mass is 480 kg. Its effective volume giving the balloon its buoyancy is V.

The balloon is floating at an equilibrium height of 100 m. When N number of sandbags are thrown out, the balloon rises to a new equilibrium height close to 150 m with its volume V remaining unchanged. If the variation of the density of air with height h from the ground is $\rho(h) = \rho_0 e^{-\frac{h}{h_0}}$, where $\rho_0 = 1.25$ kg m^{-3} and $h_0 = 6000$ m, the value of N is ________.

4 Fill in the Blanks

5. A piece of metal floats on mercury. The coefficients of volume expansion of the metal and mercury are γ_1 and γ_2 respectively. If the temperatures of both mercury and the metal are increased by an amount ΔT, the fraction of the volume of the metal submerged in mercury changes by the factor .... **[1991 - 2 Mark]**

6 MCQs with One or More than One Correct Answer

6. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity μ_0. Which of the following statements is (are) true? **[Adv. 2018]**
(a) The resistive force of liquid on the plate is inversely proportional to h
(b) The resistive force of liquid on the plate is independent of the area of the plate
(c) The tangential (shear) stress on the floor of the tank increases with μ_0
(d) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid

7 Match the Following

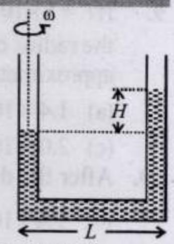
7. A person in lift is holding a water jar, which has a small hole at the lower end of its side. When the lift is at rest, the water jet coming out of the hole hits the floor of the lift at a distance d of 1.2 m from the person. In the following, state of the lift's motion is given in List-I and the distance where the water jet hits the floor of the lift is given in List-II. Match the statements from List-I with those in List-II and select the correct answer using the code given below the lists. **[Adv. 2014]**

	List - I		List - II
P.	Lift is accelerating vertically up	**1.**	$d = 1.2$ m
Q.	Lift is accelerating vertically down with an acceleration less than the gravitational acceleration	**2.**	$d > 1.2$ m
R.	Lift is moving vertically up with constant speed	**3.**	$d < 1.2$ m
S.	Lift is falling freely	**4.**	No water leaks out of the jar

Code:
(a) P-2, Q-3, R-2, S-4
(b) P-2, Q-3, R-1, S-4
(c) P-1, Q-1, R-1, S-4
(d) P-2, Q-3, R-1, S-1

10 Subjective Problems

8. A U tube is rotated about one of it's limbs with an angular velocity ω. Find the difference in height H of the liquid (density ρ) level, where diameter of the tube $d << L$. **[2005 - 2 Marks]**



9. A ball of density d is dropped on to a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density of d_L. **[1992 - 8 Marks]**
(a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.
(b) Is the motion of the ball simple harmonic?
(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid? Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

Answer Key

Topic-1 : Pressure, Density, Pascal's Law and Archimedes' Principle

1. (b) **2.** (c) **3.** (d) **4.** (d) **5.** (a) **6.** (b) **7.** (0.24) **9.** (False) **10.** (False)
11. (False) **12.** (a, c) **13.** (b, c) **14.** (a, d) **15.** (c) **16.** (b,c) **17.** (a) **18.** (c) **19.** (b) **20.** (a)
21. (a)

Topic-2 : Fluid Flow, Reynold's Number and Bernoulli's Principle

1. (a) **2.** (a) **3.** (a) **4.** (3) **5.** (c) **6.** (9) **7.** (6) **8.** (500) **9.** (c) **10.** (a)
11. (a)

Topic-3 : Viscosity and Terminal Velocity

1. (3) **2.** (a,b,c) **3.** (a, d)

Topic-4 : Surface Tension, Surface Energy and Capillarity

1. (d) **2.** (96) **3.** (6) **4.** (6) **5.** (3.74) **6.** (a,b,c) **7.** (a, c) **8.** (c) **9.** (a) **10.** (b)

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. (a) **2.** (b) **3.** (c) **4.** (4) **6.** (a, c, d) **7.** (c) **8.** $\left(\frac{\omega^2 L^2}{2g}\right)$

Hints & Solutions

Topic-1: Pressure, Density, Pascal's Law and Archimedes' Principle

1. **(b)** Pressure will be same at the same horizontal level.

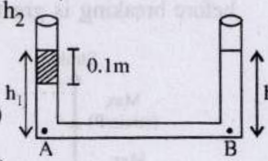
Therefore, $P_A = P_B$

$\Rightarrow P_0 + \rho_k g \times 0.1 + \rho_w g \times (h_1 - 0.1) = \rho_w g\, h_2 + \rho_0$

$\Rightarrow 80 + 1000(h_1 - 0.1) = 1000 h_2$

$\Rightarrow 80 + 1000 h_1 - 100 = 1000 h_2$

$\Rightarrow h_1 - h_2 = \dfrac{20}{1000}$



$\Rightarrow h_1 - h_2 = 0.02$ ..... (i)

Also, $h_1 - 0.1 + h_2 = 2 \times 0.29$

$\Rightarrow h_1 + h_2 = 0.68$ ..... (ii)

Solving (i) & (ii), we get

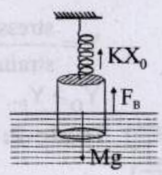
$2h_1 = 0.70$

$\Rightarrow h_1 = 0.35\text{ m}$

and $h_2 = 0.33\text{ m}$

So, $\dfrac{h_1}{h_2} = \dfrac{35}{33}$

2. **(c)** From figure, $kx_0 + F_B = Mg$



$kx_0 + \sigma \dfrac{L}{2} Ag = Mg$

[$\because$ mass = density $\times$ volume]

$\Rightarrow kx_0 = Mg - \sigma \dfrac{L}{2} Ag$

$$\Rightarrow x_0 = \frac{Mg - \dfrac{\sigma L A g}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

Hence, extension of the spring when it is in equilibrium is,

$$x_0 = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

3. **(d)** When the coin falls from the top of block into water the block moves upwards because the weight of floating body becomes less and hence l decreases. When the coil was floating, it displaces water equal to its own weight. When the coin is inside the water, it displaces water equal to its own volume. As its density is greater than that of water, it displaces more water in first case. Hence, h decreases when coin fall into the water.

4. **(d)** From Archimedes principle

Upthrust = wt. of fluid displaced

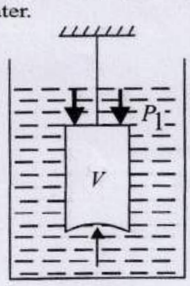
Now, $F_{bottom} - F_{top} = V\rho g$

$\Rightarrow F_{bottom} = F_{top} + V\rho g$

$= P_1 \times A + V\rho g$

$= (h\rho g) \times (\pi R^2) + V\rho g$

$= \rho g\,[\pi R^2 h + V]$



5. **(a)** Weight of cylinder = upthrust due to upper liquid + upthrust due to lower liquid.

$$D\left(\frac{A}{5} \times L \times g\right) = d\left(\frac{A}{5}\right)\left(\frac{3}{4}L\right)g + 2d\left(\frac{A}{5}\right)\left(\frac{L}{4}\right) \times g$$

$\therefore\ D = \dfrac{5d}{4}$

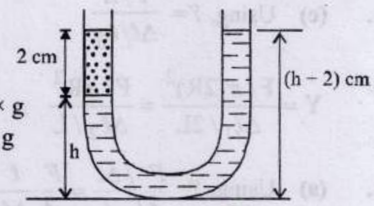
6. **(b)** We have,

$2 \times \rho_2 \times g + h \times 1.1 \times g$

$= (2 + h) \times 1.1 \times g$

$\Rightarrow 2\rho_2 = 2.2$

$\Rightarrow \rho_2 = 1.1$



7. **(0.24)** As we know, bulk modules $B = \dfrac{P}{\left(-\dfrac{dv}{v}\right)}$

$$\Rightarrow \frac{dv}{v} = -\frac{P}{B} \Rightarrow 3\left(\frac{\Delta \ell}{\ell}\right) = -\frac{P}{B}$$

$$\Delta \ell = \left(\frac{-P}{B}\right)\frac{\ell}{3} = \left(\frac{\rho g h}{B}\right)\frac{\ell}{3}$$

$$= \frac{10^3 \times 10 \times 5 \times 10^3}{70 \times 10^9} \times \frac{1}{3} = 0.238 \times 10^{-3}$$

$\therefore\ \Delta \ell = 0.238\text{ mm} \simeq 0.24\text{ mm}$

8. $K = \dfrac{-\Delta P}{\Delta V / V}$

where $\Delta P = \dfrac{Mg}{A}$ $\qquad\therefore\ -\dfrac{\Delta V}{V} = \dfrac{Mg}{AK}$

$\Rightarrow -\dfrac{(V_f - V_i)}{V_i} = \dfrac{Mg}{AK} \quad \Rightarrow \dfrac{V_i - V_f}{V_i} = \dfrac{Mg}{AK}$

$$\Rightarrow \frac{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi (R - \delta R)^3}{\frac{4}{3}\pi R^3} = \frac{Mg}{AK}$$

$$\Rightarrow \frac{R^3 - [R^3 - 3R^2 \delta R]}{R^3} = \frac{Mg}{AK} \Rightarrow \frac{\delta R}{R} = \frac{Mg}{3AK}$$

9. **False,**

When the block of ice melts, the lead shot will ultimately sink in the water. When lead shot sinks, it will displace water equal to its own volume. But when lead shot was embedded in ice, it displaced more volume of water than its own volume because $d_{lead} > d_{water}$.

$\because$ volume = $\dfrac{\text{mass}}{\text{density}}$. Hence, level of water will fall.

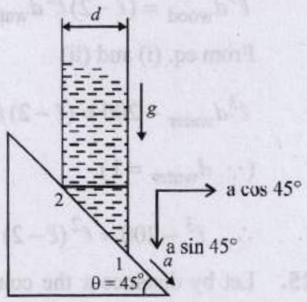
10. **False,**

When temperature is increased, the density of mercury (ρ) decreases as $\rho_t = \rho_0 (1 - \gamma t)$ and hence, the level of mercury in barometer tube (h) increases

$\because\ h\rho = \text{constant}$

11. **False,**
The level of water will remain the same in the pond.
When the man drinks some water from the pond, his weight increases and therefore the boat will sink further. And this sinking of the boat will displace the same volume of water in pond as drunk by man.

12. **(a, c)** $P_1 = P_2 - \rho\, a \cos 45^\circ\, d + \rho\,(g - a \sin 45^\circ)\, d$

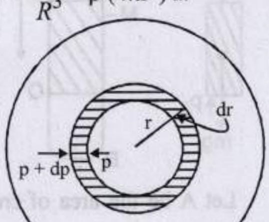


$\Rightarrow \dfrac{P_1 - P_2}{\rho g d} = 1 - \dfrac{\sqrt{2}a}{g}$

$\therefore\ \beta = 0$ for $a = g/\sqrt{2}$

and $\beta = \dfrac{\sqrt{2}-1}{\sqrt{2}}$ for $a = \dfrac{g}{2}$

13. **(b, c)** Let us consider an elemental mass dm shown in the shaded portion of the figure.



Here, $P\,4\pi r^2 - (P + dP)\,4\pi r^2 = \dfrac{GMr}{R^3}\,\rho\,(4\pi r^2)\,dr$

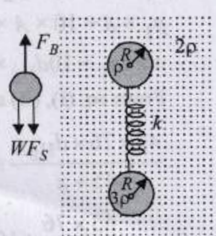
$\therefore\ -\int_0^P dP = \dfrac{GM\rho}{R^3}\int_R^r r\,dr$

$\therefore\ P = \dfrac{GM\rho}{2R^3}\left[R^2 - r^2\right]$

$$\therefore\ \frac{P\,(r = 3R/4)}{P\,(r = 2R/3)} = \frac{\left[R^2 - \frac{9R^2}{16}\right]}{\left[R^2 - \frac{4R^2}{9}\right]} = \frac{\frac{7R^2}{16}}{\frac{5R^2}{9}} = \frac{63}{80}$$

$$\text{and}\ \frac{P\,(r = 3R/5)}{P\,(r = 2R/5)} = \frac{\left[R^2 - \frac{9R^2}{25}\right]}{\left[R^2 - \frac{4R^2}{25}\right]} = \frac{16}{21}$$

14. **(a, d)** The complete system is as shown in figure.
Let x be the net elongation of the spring.
At equilibrium, for upper sphere
$W + F_S = F_B$



$\dfrac{4}{3}\pi R^3 \rho g + kx = \dfrac{4}{3}\pi R^3 (2\rho) g$

$\Rightarrow kx = \dfrac{4}{3}\pi R^3 2\rho g - \dfrac{4}{3}\pi R^3 \rho g$

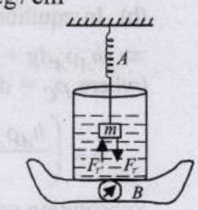
$\Rightarrow kx = \dfrac{4\pi R^3 \rho g}{3}$ or $x = \dfrac{4\pi R^3 \rho g}{3k}$

15. **(c)** Let d be the density of the material of the sphere
Weight of sphere
= Upthrust due to Hg + Upthrust due to oil

$Vdg = \dfrac{V}{2} d_{Hg}\, g + \dfrac{V}{2} d_{oil} \times g$

$\Rightarrow\ d = \dfrac{d_{Hg} + d_{oil}}{2} = \dfrac{13.6 + 0.8}{2} = 7.2\,\text{g/cm}^3$

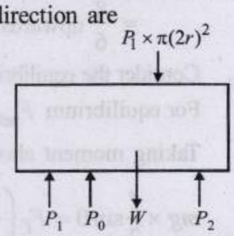
16. **(b,c)** When the block of mass m is inside the liquid an upthrust F_T will act on the mass which will decrease the reading on A i.e., will read less than 2 kg.



According to Newton's third law, to each and every action, there is equal and opposite reaction.
So by reaction an equal force will be exerted on the liquid of the beaker which will increase the reading in B i.e., will read more than 5 kg.

17. **(a)** The whole system falls freely under gravity, so $a = g$
Upthrust = $\rho V(g - a)$
$\therefore$ Upthrust = 0.

18. **(c)** Consider the equilibrium of wooden cylindrical block.
Forces acting in the downward direction are



(i) Weight of wooden cylinder

$W = \pi(2r)^2 \times h \times \dfrac{\rho}{3} \times g$

$= \pi \times 4r^2 \dfrac{h\rho}{3} g$

(ii) Force due to pressure (P_1) created by liquid of height h_1 above the wooden block
$F_1 = P_1 \times \pi\,(2r)^2 = [P_0 + h_1 \rho g] \times \pi\,(2r)^2$
(iii) Force acting on the upward direction due to pressure P_2 exerted from below the wooden block and atmospheric pressure

$F_2 = P_2 \times \pi\left[(2r)^2 - r^2\right] + P_0 \times \pi(r)^2$
$= \left[P_0 + (h_1 + h)\rho g\right] \times \pi \times 3r^2 + P_0 \pi r^2$

In situation 1, at the verge of rising the block, $F_2 = F_1 + W$

$\left[P_0 + (h_1 + h)\rho g\right] \times \left(\pi \times 3r^2\right) + \pi r^2 P_0$

$= \left[P_0 + h_1 \rho g\right] \times 4\pi r^2 + \dfrac{\pi \times 4r^2 h\rho g}{3}$ or, $h_1 = \dfrac{5h}{3}$

19. **(b)** Again considering equilibrium of wooden block.
Total downward force = total force upwards
Wt. of block + force due to atmospheric pressure = force due to pressure of liquid + Force due to atmospheric pressure

$\pi\,(16r^2)\dfrac{\rho}{3} \times g + P_0 \pi \times 16r^2 = [h_2 \rho g + P_0]\,\pi\,[(16-4)r^2] + P_0 \times 4r^2$

when h_2 = height of the water level in situation – 2 for which the block remains in its original position.

$\therefore\ h_2 = \dfrac{4}{9}h$

20. **(a)** In situation – 2 when the height h_2 of water level is further decreased, then the upward force acting on the wooden block decreases. The total force downward remains the same. This difference will be compensated by the normal reaction by the tank wall on the wooden block.

Hence the block does not moves up and remains at its original position.

21. (a) As the hydrostatic force exerted by liquid A on the cylinder from all sides they cancels out and the net value is zero.

(b) In equilibrium, buoyant force = weight of the body

$\Rightarrow h_A\rho_A Ag + h_B\rho_B Ag = (h_A + h + h_B) A \rho_C g$

(where ρ_C = density of cylinder)

$$h = \left(\frac{h_A\rho_A + h_B\rho_B}{\rho_C}\right) - (h_A + h_B)$$

Substituting value of h_A, h_B ρ_A, ρ_B and ρ_e and solving we get $h = 0.25$ cm

(c) Net acceleration, $a = \dfrac{F_{\text{Buoyant}} - Mg}{M}$

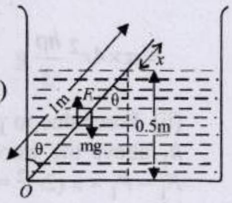
$$= \left[\frac{h_A\rho_A + \rho_B(h + h_B) - (h + h_A + h_B)\rho_C}{\rho_C(h + h_A + h_C)}\right] g$$

$= \dfrac{g}{6}$ upwards

22. Consider the equilibrium of the plank hinged at point 'O'

For equilibrium $F_{\text{net}} = 0$ and $\tau_{\text{net}} = 0$

Taking moment about point O



$mg \times \dfrac{\ell}{2}\sin\theta = F_T\left(\dfrac{\ell - x}{2}\right)\sin\theta$ .. (i)

Also F_T = wt. of fluid displaced

$= [(\ell - x)A] \times \rho_w g$ ... (ii)

And $m = (\ell A)0.5\rho_w$ ...(iii)

Where A is the area of cross section of the rod.

From eq. (i), (ii) and (iii)

$$(\ell A)0.5\rho_w g \times \frac{\ell}{2}\sin\theta = [(\ell - x)A]\rho_w g \times \left(\frac{\ell - x}{2}\right)\sin\theta$$

Here, $\ell = 1$ m

$\therefore$ $(1 - x)^2 = 0.5 \Rightarrow x = 0.293$ m

From the diagram

$\cos\theta = \dfrac{0.5}{1 - x} = \dfrac{0.5}{0.707}$ $\therefore$ $\theta = 45°$

23. Let V_ℓ, V_s = volume of liquid and stone respectively and d_ℓ, d_s = density of liquid and stone respectively

When the stones were in the boat, the weight of stones were balanced by the buoyant force.

$V_s d_s = V_\ell d_\ell$

Since, $d_s > d_\ell$ $\therefore$ $V_s < V_\ell$

Hence when stones are put in water, the level of water falls

24. Let the size or edge of cube be ℓ. When mass $m = 200$ g is on the cube of wood

$200g + \ell^3 d_{\text{wood}} g = \ell^3 d_{\text{water}} g$

$\Rightarrow$ $\ell^3 d_{\text{wood}} = \ell^3 d_{\text{water}} - 200$ ... (i)

When the mass $m = 200$ g is removed

$\ell^3 d_{\text{wood}} = (\ell - 2)\ell^2 d_{\text{water}}$ ... (ii)

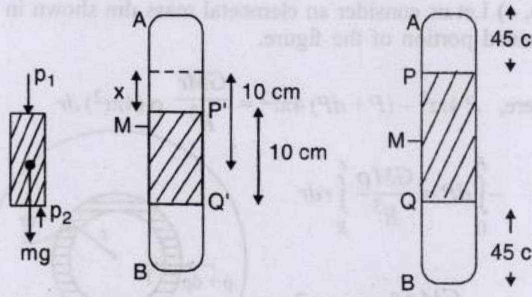
From eq. (i) and (ii)

$\ell^3 d_{\text{water}} - 200 = (\ell - 2)\ell^2 d_{\text{water}}$

$(\because d_{\text{water}} = 1)$

$\therefore$ $\ell^3 - 200 = \ell^2(\ell - 2)$ $\Rightarrow$ $\ell = 10$ cm

25. Let by distance x the column of mercury be displaced.



Let A be the area of cross-section of the tube.

Let x distance the column of mercury be displaced if the tube is held vertically.

Applying $P_1V_1 = P_2V_2$

For air present in column AP

$p \times 45 \times A = p_1 \times (45 + x) \times A$

$\Rightarrow$ $p_1 = \dfrac{45}{45 + x} \times 76 d_{\text{Hg}} \times g$ ... (i)

For air present in column QB

$p \times 45 \times A = p_2 \times (45 - x) \times A$

$\Rightarrow$ $p_2 = \dfrac{45}{45 - x} \times 76 d_{\text{Hg}} \times g$ ... (ii)

M is the mid-point of tube AB.

At equilibrium

$p_1 \times A + mg = p_2 \times A$

$p_1 \times A + 10 \times A \times d_{\text{Hg}} g = p_2 \times A$

$\Rightarrow$ $p_1 + 10 d_{\text{Hg}} \times g = p_2$ ... (iii)

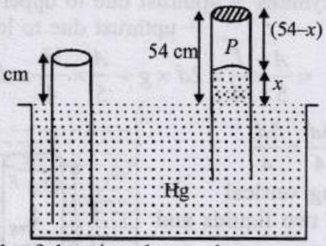
From eq. (i), (ii) and (iii)

$$\frac{45 \times 76 \times d_{\text{Hg}} g}{45 + x} + 10 d_{\text{Hg}} \times g = \frac{45}{45 - x} \times 76 \times d_{\text{Hg}} \times g$$

$\Rightarrow$ $\dfrac{45 \times 76}{45 + x} + 10 = \dfrac{45 \times 76}{45 - x}$ $\therefore$ x = 2.95 cm.

Topic-2: Fluid Flow, Reynold's Number and Bernoulli's Principle

1. (a)



Length of the air column above mercury in the tube is,

$P_f + x = P_0$

$\Rightarrow P_f = (76 - x)$

As T = cons. $\Rightarrow PV$ = cons.

$\Rightarrow P_iV_i = P_fV_f \Rightarrow (8 \times A) \times 76 = (76 - x) \times A \times (54 - x)$

$\therefore x = 38$

Thus, length of air column = 54 – 38 = 16 cm.

2. (a)

3. (a) As we know, velocity of efflux $V = \sqrt{2gh}$

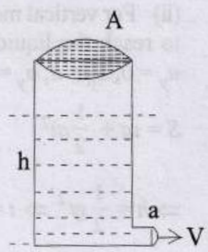
From equation of continuity $A_1V_1 = A_2V_2$

$$\sqrt{(2gy)} \times L^2 = \sqrt{(2g \times 4y)}\, \pi R^2$$

$$\Rightarrow L^2 = 2\pi R^2 \;\therefore\; R = \frac{L}{\sqrt{2\pi}}$$

4. (3) From principle of continuity $a_1v_1 = a_2v_2$

$$a\sqrt{2gh} = -A\frac{dh}{dt}$$



$$dt = -\frac{A}{a}\frac{dh}{\sqrt{2gh}}$$

$$t = \int dt = \frac{-2A}{a\sqrt{2g}}\left(\sqrt{h_f} - \sqrt{h_i}\right)$$

$$t = \frac{2A}{a\sqrt{2g}}\left(\sqrt{h_i} - \sqrt{h_f}\right)$$

For tank 1: $h_i = h, h_f = 0$

$$\therefore\; t_1 = \frac{2A}{a\sqrt{2g}}(\sqrt{h})$$

For tank 2: $h_f = \frac{16h}{9}, h_i = h + H = \frac{25h}{9}$

$$t_2 = \frac{2A\sqrt{h}}{a\sqrt{2g}}\left(\frac{5}{3} - \frac{4}{3}\right) = \frac{2A\sqrt{h}}{a\sqrt{2g}} \times \frac{1}{3}$$

$$\therefore\; \frac{T_1}{T_2} = 3$$

5. (c) Given : $v_1 = 1.0\ \text{ms}^{-1}$

$A_1 = 10^{-4}\ \text{m}^2$

v_2 = velocity of water stream at 0.15 m below the tap

$A_2 = ?$

For calculating v_2 using, $v^2 - u^2 = 2as$

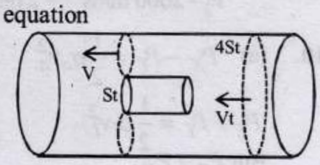
$u = 1$ m/s; $s = 1.5$ m, $a = g = 10\ \text{m/s}^2$ and $v = v_2 = ?$

$v^2 - 1 = 2 \times 10 \times 0.15 \Rightarrow v = v_2 = 2$ m/s

By equation of continuity $v_1A_1 = v_2A_2$

$$\therefore\; A_2 = \frac{v_1A_1}{v_2} = \frac{1 \times 10^{-4}}{2} = 5 \times 10^{-5} m^2$$

6. (9) Applying Bernoulli's equation



$$P_0 + \frac{1}{2}\rho v_t^2 = P + \frac{1}{2}\rho v^2$$

$$P_0 - P = \frac{1}{2}\rho(v^2 - v_t^2) \quad ...(i)$$

From equation of continuity

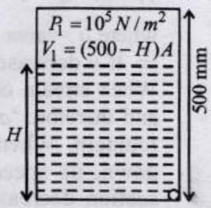
$$4S_tv_t = v \times (4S_t - S_t) = v \times 3S_t$$

$$\Rightarrow v = \frac{4}{3}v_t \quad ...(ii)$$

From eqs. (i) and (ii)

$$P_0 - P = \frac{1}{2}\rho\left(\frac{16}{9}v_t^2 - v_t^2\right) = \frac{1}{2}\rho\frac{7v_t^2}{9} = \frac{7}{2N}\rho v_t^2 \;\therefore\; N = 9$$

7. (6) Initially, pressure of air column above water $P_1 = 10^5\ \text{Nm}^{-2}$ and volume $V_1 = (500 - H)A$, where A is the area of cross-section of the vessel.

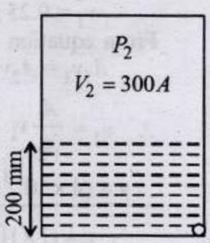


Initially

Finally, the volume of air column above water $V_2 = (500 - 200)A = 300$ A. If P_2 is the pressure of air then

$P_2 + \rho gh = P_1$

$$\therefore\; P_2 + 10^3 \times 10 \times \frac{200}{1000} = 10^5$$



Finally

$$\therefore\; P_2 = 9.8 \times 10^4 N/m^2$$

Assuming the temperature remains constant, according to Boyle's law

$P_1V_1 = P_2V_2$

$\therefore\; 10^5 \times (500 - H)A = (9.8 \times 10^4) \times 300A \Rightarrow H = 206$ mm

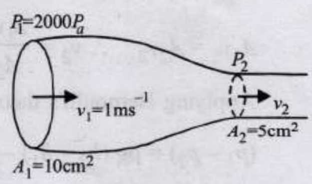
$\therefore$ Fall in height of water level due to the opening of orifice = 206 – 200 = 6 mm

8. (500) According to equation of continuity

$A_1v_1 = A_2v_2 \Rightarrow 10 \times 1 = 5 \times v_2 \Rightarrow v_2 = 2$m/s

Now, using Bernoulli's theorem

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$



$$\Rightarrow 2000 + \frac{1}{2} \times 1000 \times 1^2$$

$$= P_2 + \frac{1}{2} \times 1000 \times 2^2$$

$\therefore\; P_2 = 500\ Pa$

9. (c) Here, piston is pushed at a speed, $v_1 = 5$ m/s

Let air comes out of nozzle with a speed v_2

From principle of continuity,

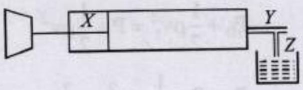
$a_1v_1 = a_2v_2$

$$\Rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2 \Rightarrow r_1^2 v_1 = r_2^2 v_2$$

$\Rightarrow (20)^2 \times 5 = (1)^2 \times v_2$

$\therefore v_2 = 2000 \text{ mms}^{-1} = 2 \text{ ms}^{-1}$

10. (a) $P_X - P_Y = \frac{1}{2}\rho_a v_a^2$



$P_Z - P_Y = \frac{1}{2}\rho_l v_l^2$

But $P_Z = P_X$

$\therefore \frac{1}{2}\rho_l v_l^2 = \frac{1}{2}\rho_a v_a^2 \Rightarrow v_l = \sqrt{\frac{\rho_a}{\rho_l}} \times v_a$

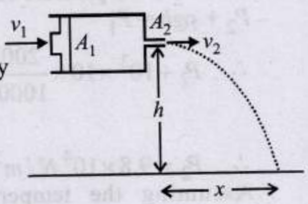
$\therefore$ Volume flow rate $\propto \sqrt{\frac{\rho_a}{\rho_l}}$

11. (a) The volume flow rate (Q) of an incompressible fluid in steady flow remains constant
From equation of continuity,
av = constant

$\therefore Q = a \times v$ = constant or, $a \propto \frac{1}{v}$

where a = area of cross-section and v = velocity
$\Rightarrow$ If v decreases a increases and vice - versa.
When stream of water moves up, its speed (v) decreases and therefore 'a' increases i.e. the water spreads out as a fountain. When stream of water from hose pipe moves down, its speed increases and therefore area of cross-section decreases.

12. Given $A_1 = \pi \times (4 \times 10^{-3} \text{ m})^2$,
$A_2 = \pi \times (1 \times 10^{-3} \text{ m})^2$
$v_1 = 0.25$ m/s



From equation of continuity
$A_1 v_1 = A_2 v_2$

$\therefore v_2 = \frac{A_1}{A_2} v_1$

$= \frac{\pi \times (4 \times 10^{-3})^2 \times 0.25}{\pi \times (1 \times 10^{-3})^2} = 4$ m/s

From $h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

Hence horizontal range

$x = v_2 \times t = v_2\sqrt{\frac{2h}{g}} = 4 \times \sqrt{\frac{2 \times 1.25}{10}} = 2\text{m}$

13. Given : $\rho = 1000 \text{ kg/m}^3, h_1 = 2\text{m}, h_2 = 5 \text{ m}$
$A_1 = 4 \times 10^{-3}\text{m}^2, A_2 = 8 \times 10^{-3}\text{m}^2, v_1 = 1$ m/s
From equation of continuity

$A_1 v_1 = A_2 v_2 \quad \therefore v_2 = \frac{A_1 v_1}{A_2} = 0.5$ m/s

Applying Bernoulli's theorem, at P and Q

$(p_1 - p_2) = \rho g (h_2 - h_1) - \frac{1}{2}\rho(v_2^2 - v_1^2)$

Now, work done/vol. by gravity forces
$= \rho g (h_2 - h_1) = 10^3 \times 9.8 \times 3 = 29.4 \times 10^3 \text{ J/m}^3$.
And

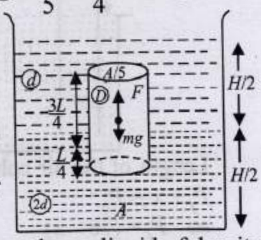
$\frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2} \times 10^3 \left[\frac{1}{4} - 1\right] = -\frac{3}{8} \times 10^3 \text{ J/m}^3$
$= -0.375 \times 10^3 \text{ J/m}^3$

$\therefore$ Work done / vol. by pressure
$P_1 - P_2 = 29.4 \times 10^3 - 0.375 \times 10^3 \text{ J/m}^3 = 29.025 \times 10^3 \text{ J/m}^3$.

14. (a) (i) Since the cylinder is in equilibrium in the liquid
$\therefore$ Weight of cylinder = upthrust due to upper liquid + upthrust due to lower liquid

$\frac{A}{5} \times L \times D \times g = \frac{A}{5} \times \frac{L}{4} \times 2d \times g + \frac{A}{5} \times \frac{3L}{4} \times d \times g$

$\Rightarrow D = \frac{2d}{4} + \frac{3d}{4} = \frac{5d}{4}$



(ii) Considering vertical equilibrium of two liquids and the cylinder.
Total pressure at the bottom of the cylinder
= Atmospheric pressure + Pressure due to liquid of density d + Pressure due to liquid of density $2d$ + Pressure due to cylinder [Weight/Area]

$P = P_0 + \frac{H}{2}dg + \frac{H}{2} \times 2d \times g + \frac{\frac{A}{5} \times L \times D \times g}{A}$

$\Rightarrow P = P_0 + \left(\frac{3H}{2} + \frac{L}{4}\right)dg \quad \left[\because D = \frac{5d}{4}\right]$

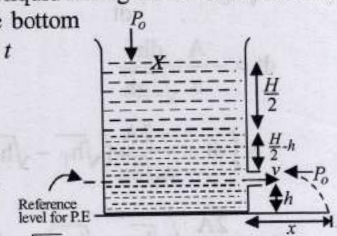
(b) (i) Applying Bernoulli's theorem

$P_0 + \left[\frac{H}{2} \times d \times g + \left(\frac{H}{2} - h\right)2d \times g\right]$

$= P_0 + \frac{1}{2}(2d)v^2 \Rightarrow v = \sqrt{\frac{(3H - 4h)}{2}g}$

(ii) For vertical motion of liquid falling from hole time taken to reach the liquid to the bottom

$u_y = 0, S_y = h, a_y = g, t_y = t$



$S = ut + \frac{1}{2}at^2$

$\Rightarrow h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

$x = v \times t = \sqrt{(3H - 4h)\frac{g}{2}} \times \sqrt{\frac{2h}{g}} = \sqrt{(3H - 4h)h}$

(iii) For finding the value of h for which x is maximum,

$\frac{dx}{dh} = 0$

$\frac{dx}{dh} = \frac{1}{2}[3H - 4h)h]^{-1/2}\{3H - 8h\}$

$\frac{1}{2}[(3H - 4h)]^{-1/2}[3H - 8h] = 0 \Rightarrow h = \frac{3H}{8}$

Hence, x will be maximum at $h = \frac{3}{4}$H

The maximum value of x

$x_m = \sqrt{\left[3H - 4\left(\frac{3H}{8}\right)\right]\frac{3H}{8}} = \sqrt{\frac{12H}{8} \times \frac{3H}{8}} = \frac{6H}{8} = \frac{3H}{4}$

Topic-3: Viscosity and Terminal Velocity

1. (3) As we know, terminal velocity

$V_T = \frac{2r^2}{9\eta}(\rho - \sigma)g$

$$\frac{V_P}{V_Q}=\frac{\frac{2r_1^2(\sigma-\rho_1)g}{9\eta_1}}{\frac{2r_2^2(\sigma-\rho_2)g}{9\eta_2}}=\frac{r_1^2(\sigma-\rho_1)}{r_2^2(\sigma-\rho_2)}\times\frac{\eta_2}{\eta_1}$$

$$=\frac{1^2}{(0.5)^2}\frac{[8-0.8]}{[8-1.6]}\times\frac{2}{3}=3$$

2. **(a, b, d)** Work done in pushing the ball to the depth d = 0.7m

$w_{all}=k_f-k_i=0$

$w_g+w_B+w_v+w_{ext}=0$

$mgd-\rho_w.\ v.gd-6\pi\eta rvd+w_{ext}=0$

$w_{ext}=\rho_w.\ v.gd-mgd$

$$=(1000\times\frac{4}{3}\times\frac{22}{7}\times\left(\frac{3}{2}\times10^{-2}\right)^3-\frac{22}{7}\times10^{-3})\,gd$$

$$w_{ext}=\frac{22}{7}\times10^{-3}\left[\frac{9}{2}-1\right]\times10\times0.7=\frac{22}{7}\times10^{-3}\times\frac{7}{2}\times7$$

$w_{ext}=77\times10^{-3}\,J=0.077J$

so option (a) is correct.

(B) When ball is released at bottom same work i.e. 0.077 J is done on ball

$$\therefore\ \frac{1}{2}mv^2=0.077$$

$$\therefore\ v=\sqrt{\frac{2\times0.077}{\frac{22}{7}\times10^{-3}}}=7m/s$$

So option (b) is correct

(C) Height $H=\frac{v^2}{2g}=\frac{49}{20}=2.45m$

so option (c) is incorrect.

Net force $F_{net}=vdg-vpg=0.11$ N

And viscous force is maximum when v = 7 m/s

$$\therefore\ (F_v)_{max}=6\pi\eta vr=6\times\frac{22}{7}\times10^{-3}\left(\frac{3}{2}\times10^{-2}\right)\times7$$

$=18\times11\times10^{-5}$N

$$\therefore\ \frac{F_{net}}{(F_v)_{max}}=\frac{0.11}{18\times11\times10^{-5}}=\frac{500}{9}$$

so, option (d) is correct

3. **(a, d)** Since string is taut, $\rho_1<\sigma_1$ and $\rho_2<\sigma_2$

For floating, net weight of system = net upthrust

$(\rho_1+\rho_2)V_g=(\sigma_1+\sigma_2)V_g$

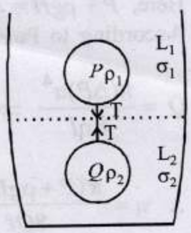
Upward terminal velocity

$$V_P=\frac{2r^2(\sigma_2-\rho_1)g}{9\eta_2}$$

Where r is radius of sphere.

Downward terminal velocity

$$V_Q=\frac{2r^2(\rho_2-\sigma_1)g}{9\eta_1}$$



$$\therefore\ \left|\frac{V_P}{V_Q}\right|=\frac{\eta_1}{\eta_2}$$

$(\because\ \rho_1-\sigma_2=\sigma_1-\rho_2)$

Again $V_P\,.\,V_Q<0$ *i.e.*, negative as V_P and V_Q are opposite to each other.

Topic-4: Surface Tension, Surface Energy and Capillarity

1. **(d)**

2. **(96)** Since temperature remain unchanged i.e., T = constant so $P_1V_1=P_2V_2$

$$V_1=\frac{4}{3}\pi R_1^3\text{ and }V_2=\frac{4}{3}\pi R_2^3$$

Excess pressure, $P_1=P_0+\Delta P_1$ and $\Delta P_1=\frac{4T}{R_1}$

$$P_2=\frac{8P_0}{27}+\Delta P_2\text{ and }\Delta P_2=\frac{4T}{R_2}$$

$$\therefore\ (P_0+\Delta P_1)\times\frac{4}{3}\pi R_1^3=\left(\frac{8P_0}{27}+\Delta P_2\right)\times\frac{4}{3}\pi R_2^3$$

$$\text{or, }(P_0+\Delta P_1)\left(\frac{4T}{\Delta P_1}\right)^3=\left(\frac{8P_0}{27}+\Delta P_2\right)\left(\frac{4T}{\Delta P_2}\right)^3$$

$[\because\ \Delta P_1<<P_0]$

$$\frac{P_0}{(\Delta P_1)^3}\approx\frac{8P_0}{27}\times\frac{1}{(\Delta P_2)^3}$$

$$\text{or,}\quad\Delta P_2=\frac{2}{3}\Delta P_1=\frac{2}{3}\times(144\text{Pa})$$

$\therefore\quad\Delta P_2=96$ Pa

3. **(6)** $\frac{4}{3}\pi R^3=k\times\frac{4}{3}\pi r^3\quad\therefore\quad R=K^{1/3}r$

$\Delta U=S[k\times4\pi r^2-4\pi R^2]$

$$\therefore\ \Delta U=4\pi s\left[k\times\frac{R^2}{k^{2/3}}-R^2\right]=4\pi SR^2\left[k^{1/3}-1\right]$$

$\therefore\ \Delta U=4\pi SR^2\left[10^{\alpha/3}-1\right]\ [\because\ K=10^{\alpha}]$

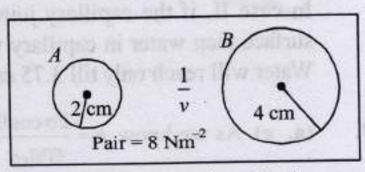
$$\therefore\ 10^{-3}=4\pi\times\frac{0.1}{4\pi}\times(10^{-2})^2\left[10^{\alpha/3}-1\right]$$

$\therefore\ 10^2=10^{\alpha/3}-1$ Neglecting

$10^2=10^{\alpha/3}\ \Rightarrow\ \frac{\alpha}{3}=2\quad\therefore\ \alpha=6$

4. **(6)** Pressure,

$$P_A=P_{air}+\frac{4T}{R_A}$$



($\because$ Excess pressure due to surface tension (*T*) in soap bubble = $4T/r$)

$$P_A=8+\frac{4T}{R_A}=\frac{4\times0.04}{0.02}+8\ \Rightarrow\ P_A=16N/m^2$$

Similarly, $P_B=8+\frac{4T}{R_B}=8+\frac{4\times0.04}{0.04}=12\text{ N/m}^2$

According to ideal gas equation, $P_V=nRT$

$P_AV_A = n_ART_B \Rightarrow 16\times\frac{4}{3}\pi(0.02)^3 = n_ART_A$ ...(i)

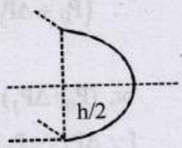
$P_BV_B = n_BRT_B \Rightarrow 12\times\frac{4}{3}\pi(0.04)^3 = n_BRT_B$ ... (ii)

Dividing eq. (ii) by (i)

$$\frac{12\times\frac{4}{3}\pi(0.04)^3}{16\times\frac{4}{3}\pi(0.02)^3} = \frac{n_B}{n_A} \quad [\because T_A = T_B]$$

$$\therefore \frac{n_B}{n_A} = 6$$

5. **(3.74)** According to question, the water above the rim as a disc of thickness 'h' having semicircular edges.
r = h/2
Pressure at the bottom of disc = pressure due to surface tension



$$\rho gh = T\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

$R_1 >>> R_2 \therefore \frac{1}{R_1} <<< \frac{1}{R_2}$ and $R_2 = h/2$

$$\therefore \rho gh = T\left[\frac{1}{R_1}+\frac{1}{R_2}\right] = T\left[0+\frac{1}{h/2}\right] = \frac{2T}{h}$$

$$\Rightarrow h^2 = \frac{2T}{\rho g} \Rightarrow h = \sqrt{\frac{2T}{\rho g}} = \sqrt{\frac{20\times0.07}{10^3\times10}} = \sqrt{\frac{14\times100}{10^4\times100}}$$

$\therefore h = \sqrt{14}$ mm $= 3.741$

6. **(a, b, c)** **For case I**

$$h_1 = \frac{2T\cos\theta_1}{r\rho g} = \frac{2\times0.75\times\cos0^\circ}{2\times10^{-4}\times1000\times10} = 7.5\text{cm}$$

For case II

$$h_2 = \frac{2T\cos\theta_2}{r\rho g} = \frac{2\times0.75\times\cos60^\circ}{2\times10^{-4}\times1000\times10} = 3.75\text{cm}$$

The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus will be different for both cases.

In case II, if the capillary joint is 5 cm above the water surface then water in capillary will not reach the interface. Water will reach only till 3.75 cm.

7. **(a, c)** As we know $h = \frac{2\sigma\cos\theta}{r\rho g_{eff}}$

As 'r' increases, h decreases $h \propto \frac{1}{r}$

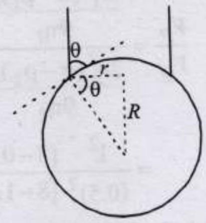
[all other parameters remaining constant]

Also $h \propto \sigma$

Further if lift is going up with an acceleration 'a' then $g_{eff} = g + a$. As g_{eff} increases, 'h' decreases.

Also $h \propto \cos\theta$ not $h \propto \theta$

8. **(c)** Vertical force due to surface tension



$F_V = (T\cos\theta)\times 2\pi r$

$$= T\left(\frac{r}{R}\right)\times 2\pi r = \frac{2\pi r^2 T}{R}$$

$$\left[\because \cos\theta = \frac{r}{R}\right]$$

9. **(a)** When the drop is about to detach from the dropper

Weight = vertical force due to surface tension ($mg = F_V$)

$$\therefore \frac{4}{3}\pi R^3\rho g = \frac{2\pi r^2 T}{R}$$

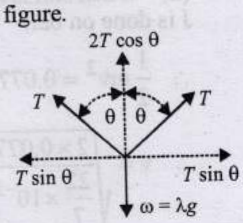
$$\Rightarrow R^4 = \left(\frac{3}{2}\frac{r^2T}{\rho g}\right) = \frac{3}{2}\times\frac{(5\times10^{-4})^2\times0.11}{1000\times10} = 4.12\times10^{-12}$$

or, $R = 1.42\times10^{-3}$ m

10. **(b)** Surface energy = T × 4πR²

$$= 0.11\times4\times\frac{22}{7}\times(1.42\times10^{-3})^2 = 2.7\times10^{-6}\text{ J}$$

11. For equilibrium of wire in vertical direction, from free body diagram of the wire shown in the figure.



$\therefore 2Tl\cos\theta = \lambda lg$

or $2T\cos\theta = \lambda g$

$\cos\theta = \frac{y}{a}$

while $y << a$

$$\therefore \frac{2T\times y}{a} = \lambda g \quad\text{or}\quad T = \frac{\lambda ag}{2y}$$

12. When the tube is not there, using Bernoulli's theorem

$$P + P_0 + \frac{1}{2}\rho v_1^2 + \rho gH = \frac{1}{2}\rho v_0^2 + P_0$$

$$\Rightarrow P + \rho gH = \frac{1}{2}\rho(v_0^2 - v_1^2)$$

From equation of continuity $A_1v_1 = A_2v_2$

$$v_1 = \frac{A_2v_0}{A_1} \quad [\text{here } v_2 = v_0]$$

$$\therefore P + \rho gH = \frac{1}{2}\rho\left[v_0^2 - \left(\frac{A_2}{A_1}v_0\right)^2\right]$$

$$P + \rho gH = \frac{1}{2}\rho v_0^2\left[1-\left(\frac{A_2}{A_1}\right)^2\right]$$

Here, $P + \rho gH = \Delta P$

According to Poisseuille's equation

$$Q = \frac{\pi(\Delta P)a^4}{8\eta l} \Rightarrow \eta = \frac{\pi(\Delta P)a^4}{8Q\ell}$$

$$\therefore \eta = \frac{\pi(P+\rho gH)a^4}{8Q\ell} = \frac{\pi}{8Q\ell}\times\frac{1}{2}\rho v_0^2\left[1-\left(\frac{A_2}{A_1}\right)^2\right]\times a^4$$

$$\eta=\frac{\pi}{8Ql}\times\frac{1}{2}\rho v_0^2\left[1-\frac{d^4}{D^4}\right]\times a^4\quad\left[\because\frac{A_2}{A_1}=\frac{d^2}{D^2}\right]$$

13. The bubble will separate from the ring, when the force due to air = $\rho A V^2$ in the bubble equals the surface tension force $4T/R$ inside the bubble.

$$\therefore\quad \rho A v^2=\frac{4T}{R}\times A\quad\Rightarrow R=\frac{4T}{\rho v^2}$$

Radius at which bubbles separate from the ring $R=\frac{4T}{\rho v^2}$

Topic-5: Miscellaneous (Mixed Concepts) Problems

1. **(a)** Given : $\frac{1}{\rho}\frac{d\rho}{dt}$ = constant

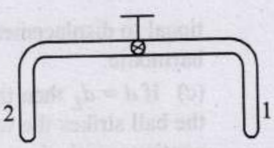
$$\therefore\quad\frac{4\pi R^3}{3m}\frac{d}{dt}\left[\frac{m}{\frac{4}{3}\pi R^3}\right]=\text{constant}$$

$$\Rightarrow\quad R^3\frac{d}{dt}(R^{-3})=\text{constant}$$

$$\Rightarrow\quad R^3(-3R^{-4})\frac{dR}{dt}=\text{constant}\quad\therefore\quad\left|\frac{dR}{dt}\right|\propto R$$

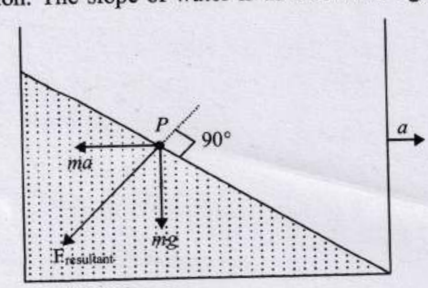
2. **(b)** We know that excess pressure in a soap bubble is inversely proportional to its radius. *i.e.*, $P=\frac{4T}{r}$

The soap bubble at end 1 has small radius as compared to the soap bubble at end 2 (given). Therefore excess pressure at 1 is more.



Hence, air flows from end 1 to end 2 and the volume of soap bubble at end 1 decreases.

3. **(c)** It force acts on water towards right, when a vessel containing water is given a constant acceleration a towards the right. As an action the water exerts a force on the wall of vessel towards right. As a reaction the wall pushes the water towards left. According to Newton's 3rd law of motion. The slope of water is as shown in figure.



P is a particle of water.

The surface of water should set it self at 90° to $F_{resultant}$.

4. **(4)** According to question, variation of the density of air with height h from the ground.

$$\rho(h)=\rho_0 e^{-\frac{h}{h_0}}$$

At h = 100 m, mass m = 480 kg

$\therefore$ mg = $V g\rho_{(100)}$

$\Rightarrow$ 480 × g = $V\rho_1 g$

A h = 150 m, mass m = (480 – N)kg

$\therefore$ mg = $V g\rho_{(150)}$

$\Rightarrow$ (480 – N)g = $V\rho_2 g$

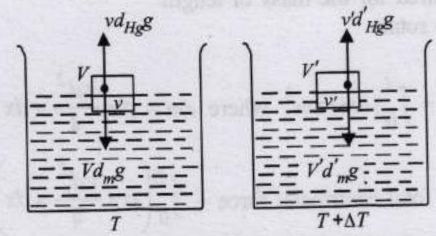
$$\frac{480-N}{480}=\frac{\rho_2}{\rho_1}\Rightarrow\left(1-\frac{N}{480}\right)=\frac{e^{-h_2/h_0}}{e^{-h_1/h_0}}=e^{\frac{h_1-h_2}{h_0}}=e^{\frac{50}{6000}}$$

$$\Rightarrow 1-\frac{N}{480}=1-\frac{50}{6000}\ \therefore N=\frac{50\times480}{6000}=4$$

5. The condition for floatation, $v d_{Hg} g = V d_m g$

Fraction of volume of metal submerged in mercury

$$=\frac{v}{V}=\frac{d_m}{d_{Hg}}=x\,(\text{say})$$



In second case, when temperature is increased by ΔT.

$$v'd'_{Hg}g=V'd'_m g$$

$$\Rightarrow\quad\frac{v'}{V'}=\frac{d'_m}{d'_{Hg}}=\text{Fraction of volume of metal submerged}$$

in mercury = x' (say)

$$\therefore\frac{x'}{x}=\frac{d'_m\times d_{Hg}}{d'_{Hg}\times d_m}=\frac{d'_m\times d'_{Hg}(1+\gamma_2\Delta T)}{d'_{Hg}\times d'_m(1+\gamma_1\Delta T)}=\frac{(1+\gamma_2\Delta T)}{(1+\gamma_1\Delta T)}$$

$$=(1+\gamma_2\Delta T)(1+\gamma_1\Delta T)^{-1}$$

$$=(1+\gamma_2\Delta T)(1-\gamma_1\Delta T)=1+(\gamma_2-\gamma_1)\Delta T-\gamma_1\gamma_2(\Delta T)^2$$

$$\frac{x'}{x}-1=(\gamma_2-\gamma_1)\Delta T\quad\Rightarrow\quad\frac{x'-x}{x}=(\gamma_2-\gamma_1)\Delta T$$

6. **(a, c, d)** $F=-\eta A\left(\frac{dv}{dx}\right)$ or $|F|=\eta A\,\frac{u_0}{h}$

where $\frac{u_0}{h}$ = velocity gradient $\left(\frac{dv}{dx}\right)$

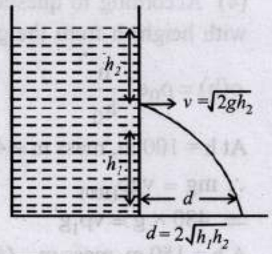
$F\propto\eta$; $F\propto A$; $F\propto\mu_0$ and $F\propto\frac{1}{h}$

7. **(c)** Horizontal distance,

$d=v\times t$

$$= \sqrt{2gh_2} \times \sqrt{\frac{2h_1}{g}}$$

$$= 2\sqrt{h_1 h_2}$$



If $g_{eff} > g$

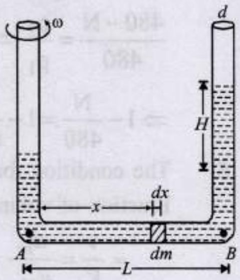
$g_{eff} = g$

$g_{eff} < g$

In all the three cases $d = 2\sqrt{h_1 h_2} = 1.2$ m

If $g_{eff} = 0$, then no water leaks out as there will be no pressure difference.

8. $\left(\frac{\omega^2 L^2}{2g}\right)$ Let us consider a small elemental mass dm at a distance x from A as shown in the figure. Centripetal force required for the mass dm to rotate $= (dm)\, x\omega^2$



$\therefore$ Total centripetal force required for the mass of length L to rotate

$$= \int_0^L (dm)\, x\omega^2 \quad \text{where } dm = \rho \times \frac{\pi d^2}{4} \times dx$$

$$\therefore \text{ Total centripetal force} = \int_0^L \left(\rho \times \frac{\pi d^2}{4} \times dx\right) \times \left(x\omega^2\right)$$

$$= \rho \times \frac{\pi d^2}{4} \times \omega^2 \int_0^L x\, dx = \rho \times \frac{\pi d^2}{4} \times \omega^2 \times \frac{L^2}{2}$$

This centripetal force is provided by the weight of liquid of height H

Weight of liquid of height H

$$= \frac{\pi d^2}{4} \times H \times \rho \times g$$

$$\therefore \frac{\pi d^2}{4} \times H \times \rho \times g = \rho \times \frac{\pi d^2}{4} \times \frac{\omega^2 \times L^2}{2} \quad \text{or, } H = \frac{\omega^2 L^2}{2g}$$

9. (a) Since density of ball (d) < density of liquid (d_L) upthrust > weight.

$\therefore$ The ball falls with retardation inside liquid.

$$\therefore \text{ Retardation} = \frac{\text{upthrust} - \text{weight}}{\text{mass}}$$

$$\text{or } a = \frac{Vd_L g - Vdg}{Vd} \quad \text{or } a = \left(\frac{d_L - d}{d}\right) g \quad \text{...(i)}$$

Let the ball come to rest inside liquid in time t.

$$\therefore t = \frac{v}{a} = \frac{gt_1}{2a}$$

[$\because$ Velocity of the ball just before it collides with liquid, $V = gt_1/2$]

$$\text{or } t = \frac{gt_1}{2\left(\frac{d_L - d}{d}\right) g} = \frac{dt_1}{2(d_L - d)} \quad \text{...(ii)}$$

$\therefore$ The time of rise to liquid surface $= t$.

$\because$ t_2 = Time taken by the ball to come back to the position from which it was released.

or $t_2 = t_1 + 2t$ substitute t from (ii)

$$\text{or } t_2 = t_1 + \frac{2 \times dt_1}{2(d_L - d)} \quad \text{or } t_2 = t_1\left[1 + \frac{d}{(d_L - d)}\right]$$

$$\text{or } t_2 = \frac{t_1 d_L}{d_L - d}$$

(b) Since the retardation $= \left(\frac{d_L - d}{d}\right) g$ is not proportional to displacement, the motion of the ball is not simple harmonic.

(c) If $d = d_L$ then the retardation = retardation = 0. Since the ball strikes the water surface with some velocity, it will continue with the same velocity $v = gt_1/2$ in downward direction.